A Fuzzy Decision Model for Conceptual Design

A.M.M. Sharif Ullah*

Department of Mechanical Engineering, UAE University, PO Box 17555, Al Ain, United Arab Emirates

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ABSTRACT

In conceptual design, there are decision problems wherein the information cannot be assessed precisely in a quantitative manner, but may be assessed in a qualitative manner. This necessitates a linguistic description on a decision problem in conceptual design. Based on this viewpoint a new fuzzy decision model for selecting the preferred conceptual design from a set of alternatives is presented. The model uses a structured form of linguistic information called “General-Opinion and Desire,” or GD. The first part of GD is a set of propositions that encode the general opinion about a conceptual design alternative for a criterion using a set of quantifiers. The other part of GD is a special proposition that encodes the desire, or requirement, for a preferred conceptual design alternative. If appropriate truth-values, taken from the interval [0, 1], are assigned to these propositions, one can determine how clearly the conceptual design alternative under consideration is known (certainty compliance) and how desirable the alternative is (desire compliance). Two functions are developed to measure the certainty and desire compliances. An aggregation function is also developed to aggregate the payoffs of certainty and desire compliances, in case the selection is made by using a set of criteria. Using the case of a real-life conceptual design problem, it is shown that the proposed decision model is useful in selecting the preferred conceptual design from a given set of alternatives. © 2005 Wiley Periodicals, Inc. Syst Eng 8: 296–308, 2005

Key words: conceptual design; linguistic information; fuzzy logic; multiple-criteria decision-making

1. INTRODUCTION

Conceptual design [Pahl and Beitz, 1996; Hsu and Liu, 2000; Wang et al., 2002] heavily affects embodiment and detailed design and around 80% of the total design and manufacturing cost of a product or a system is determined by the preferred conceptual design [Wood and Agogino, 1996]. Two types of computational ap-
approaches are found in the literature for conceptual design. One is based on computational synthesis where low-level building blocks having definite scientific relationships are synthesized to achieve high-level functionalities. See Umeda et al. [1996], Hsu and Woon [1998], Antonsson and Cagan [2001], and Yoshioka et al. [2004] for more details on computational synthesis based conceptual design. The other approach is based on decision-making, which is the focus of this paper. In this approach, the designers first propose a set of plausible conceptual design alternatives, and then use a formal decision model to select the preferred alternative [Thurston, 1991; Verma, Smith, and Fabrycky, 1999; Wang, 2001; Ullah, 2004; Smith and Verma, 2004].

The input information for decision-based conceptual design is by nature linguistic, because it (input information) is originated from some human-intelligence intensive activities rather than from some scientific analyses [Ahmed, 2003]. For example, consider the conceptual design scenario shown in Figure 1 of a chair with moveable back and leg supports for comfortable and efficient office-work. In this scenario, a group of designers is seen expressing their personal preferences, seeking advice for other experts, retrieving information from the previously solved similar design problems. While performing such human-intelligence intensive activities three plausible conceptual design alternatives are proposed as shown in Figure 1. Each alternative will be evaluated based on some linguistically expressed criteria and requirements as shown in Figure 1 under the item “general requirements.” (This design problem will be discussed in more detail in Section 5.)

The above scenario suggests that in conceptual design there are decision situations wherein the information cannot be assessed precisely in a quantitative manner, but may be assessed in a qualitative manner. This necessitates decision models capable of handling (structured) linguistic information [Verma, Smith, and Fabrycky, 1999; Wang, 2001; Ullah, 2004; Smith and Verma, 2004]. In the presently available decision models, the determination of the importance of criteria is a cumbersome task for a user to perform [Shamsuzzaman, Ullah, and Bohez, 2003; Yager, 2004]. Moreover, the computation of underlying decision information is also not so easy particularly when decision information is expressed by using fine classes (very high, high, moderate, low, very low) for some criteria and is expressed by using coarse classes (high, moderate, low) for some other criteria [Herrera, Herrera-Viedma, and Martinez, 2000]. In addition, in conceptual design the requirements are vaguely defined requiring a preprocessing of the requirements before setting the decision problem into a decision model. Therefore, further investigations are needed to develop

Figure 1. A scenario of decision-based conceptual design.
more human-friendly decision models capable of handling linguistic information as straightforwardly as possible. As a contribution to such investigations, this paper offers a new fuzzy decision model for selecting the preferred conceptual design from a given set of alternatives. In the proposed model, a piece of structured linguistic information is used to provide a means to encode the humanistic judgment, preference, intuition, etc. This piece of information is hereinafter referred to as “General Opinion and Desire,” or GD. The general opinion part of GD provides a means to encode “general opinion” on a set of conceptual design alternatives for a criterion using a set of quantifiers. The desire part of GD, on the other hand, provides a means to encode “desire” or “requirements” of a preferred conceptual design.

The structure of this paper is as follows: Section 2 describes the preliminary considerations that help define GD. Section 3 presents the formal structure of GD and the functions needed to compute it. Section 4 shows how a decision-making process, in general, should be carried out using GD. Section 5 describes how GD is applied in selecting the preferred conceptual design alternative from a given set of alternatives under multiple criteria. Before concluding the paper, a brief discussion is presented showing the main features of the GD base decision model and the avenues for further improvement.

2. PRELIMINARIES

It is accepted that fundamental facets of human-intelligence intensive activities are “partiality” and “granularity” [Zadeh, 1997, 2001, 2004], which are articulated by using linguistic information. Partiality means tolerance to the partial truth, partial understanding, and alike. Granularity means formation of granules, where granules are words as labels of values and attributes. See Zadeh [1997, 2001, 2004] for more details. GD should underlie partiality and granularity as straightforwardly as possible. Based on such contemplation, this section describes some mathematical settings needed in defining the formal structure of GD. First, an informal structure of GD is introduced in Table I so that the semantics of formal structure of GD (introduced in the next section) remains unambiguous.

As seen from Table I, the general opinion part of GD uses a sequence of propositions (p1, p2, p3, p4) providing a means to encode the general opinion about an alternative for a criterion based on some quantifiers. Each proposition has a truth-value expressed by a number in the interval [0, 1] or by a linguistic expression.

<table>
<thead>
<tr>
<th>Table I. Informal Settings of GD</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Opinion = (p1, p2, p3, p4)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Criterion</th>
<th>...</th>
<th>Alternative</th>
<th>...</th>
<th>Quantifier</th>
<th>...</th>
<th>Truth-Value</th>
<th>Proposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safety of Design X</td>
<td>is Poor</td>
<td>is</td>
<td>Absolutely False</td>
<td>p1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Safety of Design X</td>
<td>is Fair</td>
<td>is</td>
<td>Quite False</td>
<td>p2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Safety of Design X</td>
<td>is Good</td>
<td>is</td>
<td>Mostly True</td>
<td>p3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Safety of Design X</td>
<td>is Excellent</td>
<td>is</td>
<td>Somewhat True</td>
<td>p4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Criterion</th>
<th>...</th>
<th>Index</th>
<th>Quantifier</th>
<th>Truth Value</th>
<th>Proposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safety</td>
<td>should be</td>
<td>at least</td>
<td>Good</td>
<td>max(somewhat true, mostly true)</td>
<td>p4</td>
</tr>
</tbody>
</table>

“...” = syntactic phrases
The number of propositions in the following notations can be used to denote the alternative (e.g., the expressions shown in Table I). Therefore, each proposition in the general opinion part is composed of the following elements: criterion, alternative, quantifier, truth-value, and syntactic phrases.

The desire, or requirement, part of GD expresses the desire, or requirement. A single proposition \( p_d \) is used to do this. This proposition is composed of the following elements: criterion, index, quantifier, truth-value, and syntactic phrases. Note that the criterion and the quantifier associated with the desire proposition are taken from the propositions of the general opinion part. As such, the truth-value of the desire proposition is calculated from the truth-values of the general opinion propositions using fuzzy logic.¹

To develop the formal structure of GD based on the above-mentioned informal structure, it is necessary to define some mathematical entities. The remainder of this section describes the required mathematical entities.

Consider an \( n \)-tuple vector of propositions,

\[
P = (p_1, \ldots, p_n), \quad n \geq 2.
\]  (1)

Each proposition \( p_i \in P \) consists of four elements: (1) a criterion, (2) an alternative, (3) a quantifier, and (4) few syntactic phrases. For example, if \( p_1 \) = “Safety of Design X is excellent,” then “Safety” is the criterion, “Design X” is the alternative, “excellent” is the quantifier, and “of, is” are the syntactic phrases. The alternative \( A \), criterion \( C \), and syntactic phrase \( SP \) are same for all propositions in \( P \), i.e., \( A = A_1 = \ldots = A_n \), \( C = C_1 = \ldots = C_n \), \( SP = SP_1 = \ldots = SP_n \). But, each proposition in \( P \) has a unique quantifier \( Q_i \), i.e., \( Q_1 \neq \ldots \neq Q_n \). As such, \( p_i = (A, C, Q_i, SP) \), \( i = 1, \ldots, n \). The following notations can be used to denote the alternative, the criterion, the vector of quantifiers, and the number of propositions in \( P \):

\[
\text{Alternative}(P) = A, \\
\text{Criterion}(P) = C, \\
\text{Quantifier}(P) = (Q_1, \ldots, Q_i, \ldots, Q_n), \\
\text{Size}(P) = n \ (\geq 2).
\]  (2)

For example, consider the following \( P_1 = (p_1 = \text{Safety of Design X is Poor}, p_2 = \text{Safety of Design X is Fair}, p_3 = \text{Safety of Design X is Average}, p_4 = \text{Safety of Design X is Good}, p_5 = \text{Safety of Design X is Excellent}). \) In \( P_1 \), Alternative\( (P_1) = \text{Design X} \), Criterion\( (P_1) = \text{Safety} \), Quantifier\( (P_1) = (\text{Poor, Fair, Average, Good, Excellent}) \), Size\( (P_1) = 5 \).

The above notations can be used for individual proposition \( p_i \) if needed. In this case, Quantifier\( (.) \) denotes a single quantifier instead of denoting a vector of quantifiers.

Now, appropriate truth-values drawn from the interval \([0, 1]\) can be assigned to define how true or false the propositions in \( P \) are. This means

\[
p_i = TV_i \quad \forall TV_i \in [0, 1]. \]  (3)

This leads to an operation, TruthValue, as follows:

\[
\text{TruthValue}(p_i) = TV_i. \]  (4)

For example, consider the following truth-values are assigned to the propositions in \( P_1 \), \( \text{TruthValue}(p_1) = 0, \text{TruthValue}(p_2) = 0, \text{TruthValue}(p_3) = 0.1, \text{TruthValue}(p_4) = 0.8, \text{TruthValue}(p_5) = 0.2 \). The sense is that Design X is considered to be mostly good in Safety. As such, if someone is looking for a design with good safety features, then Design X is an acceptable alternative. However, if someone is looking for a design with fair safety features, Design X is not an acceptable alternative because “Safety of Design X is fair” is absolutely false.

The above exemplification implies that an additional proposition can be used along with \( P \) to express the requirement, or desire of a preferable design. A proposition \( P_d \in \{P_{d1}, P_{d2}\} \) can be used to express the desire, or requirement. The definition of \( P_d \) (i.e., \( P_{d1} \) and \( P_{d2} \)) is as follows:

\[
P_{d1} = (\text{Criterion}(P), \text{Index}, \text{Quantifier}(p_i), \text{SP}_d) \\
P_{d2} = (\text{Criterion}(P), \text{or}, \text{Quantifier}(p_i), \\
\quad \text{Quantifier}(p_{k+1}), \text{SP}_d)
\]  (5)

In (5), \( \text{Index} \in \{\text{none, at best, at least, somewhat, more or less}\}; \text{Quantifier}(p_i), \text{Quantifier}(p_{k+1}) \). Quantifier\( (p_{k+1}) \) \( \in \text{Quantifier}(P); \exists j \in \{1, \ldots, n\}; \text{and} \exists k \in \{1, \ldots, n - 1\} \). Note that \( P_{d2} \) has a definite index, “or,” whereas \( P_{d1} \) has an index taken from the set “Index.” Also, \( P_{d1} \) has a single quantifier, whereas \( P_{d2} \) has two quantifiers. \( \text{SP}_d \) is a set of syntactic phrases. For example, recall the case of \( P_1 \) where Criterion\( (P_1) = \text{Safety} \), Quantifier\( (P_1) = (\text{Poor, Fair, Average, Good, Excellent}) \). Now, if the desire, or requirement, is expressed by the following proposition, “Safety should be more or less good,” according to (5), this formulation refers to \( P_{d1} = P_{d1} = (\text{Criterion}(P_1) = \text{Safety}, \text{Quantifier}(p_i) = \text{good (} \in \text{Quantifier}(P_1)), \text{Index} = \text{more or less}, \text{SP}_d = \text{"should be"}) \).

The truth-value of \( P_d \) depends on the truth-values already assigned to the propositions in \( P \). In other
words, TruthValue(Pₜ₅) is a function of TVₜ. This implies the following expression:

\[
\text{Truth Value}(Pₜ₅) \in \{\text{Truth Value}(Pₜ₁), \text{Truth Value}(Pₜ₂)\},
\]

where

\[
\text{Truth Value}(Pₜ₁) =
\begin{cases}
TV_j & \text{if Index = "None,"} \\
\max(TV_j, TV_{j+1}, \ldots, TV_n) & \text{if Index = "at least,"} \\
\sqrt{TV_j} & \text{if Index = "somewhat,"} \\
\text{or "more or less,"}
\end{cases}
\]

\[
\text{Truth Value}(Pₜ₂) = \max(TV_k, TV_{k+1}).
\]

The operations in (6) correspond to the commonly used operation of fuzzy logic [Dubois and Prade, 2000].

For example, consider that P₁ₜ₁ = P₁ₜ₅ = (Criterion(P₁), Index = at least, Quantifier(p₅), SPₜ₅ = should be) = “Safety should be at least good.” According to (6), the truth-value of P₁ₜ₁ is TruthValue(P₁ₜ₁) = max(TVₜ₁, TVₜ₅) = max(0.8, 0.2) = 0.8. Again, if P₁ₜ₂ = P₁ₜ₅ = (Criterion(P₁), Index = somewhat, Quantifier(p₅), SPₜ₅ = should be) = “Safety should be somewhat good.” In this case, TruthValue(P₁ₜ₂) = √0.8 = 0.894.

However, it may not be an easy task to assign numerical truth-values, TVₜ. As a convenient alternative, one can consider assigning linguistic truth-values (as is done in Table I). In this case, a vector of linguistic truth-values is assigned to the propositions in Table I). In this case, a vector of linguistic truth-values is assigned to the propositions in Table I. Before it can be used. Later these fuzzy numbers can be computed to assign a numerical truth-value like TVₜ, replacing the linguistic truth-values used. To achieve this, the following logical operations can be used.

\[
p_i \models LT_i, LT_i \rightarrow E(LT_i), \quad p_i \models E(LT_i)
\]

so that

\[
LT_i = \{(x, \mu_{LT_i}(x))|x \in [0, 1], \mu_{LT_i} \in [0, 1]\}, \quad \forall i \in \{1, \ldots, n\}
\]

\[
E(LT_i) = \frac{\int x \cdot \mu_{LT_i}(x) \, dx}{\int \mu_{LT_i}(x) \, dx} \quad \forall i \in \{1, \ldots, m\},
\]

\[
E(LT_q) < E(LT_{q+1}), \quad \forall q \in \{1, \ldots, m-1\}. \quad (7)
\]

The expressions in (7) simply refers to the fact that if someone prefers to use linguistic truth-values, then the expected values of the used linguistic truth-values, according to the centroid method, denoted by E(LTₜ), take the place of TVₜ. Based on the formulation in (7), a function, denoted by LinguisticTruthValue(pₗ) = E(LTₜ), can be considered to express the fact that a linguistic truth-value is converted into a numerical truth-value.

For example, consider the following linguistic truth-values: LT₁ = (Absolutely True (AT), Mostly True (MT), Quite True (QT), Probably True (PT), Somewhat True (ST), Not Sure (NS), Somewhat False (SF), Probably False (PF), Quite False (QF), Mostly False (MF), Absolutely False (AF)). Here, m = 11. The elements of LT₁ can be defined by trapezoidal fuzzy numbers [in general (a, b, c, d)] or by triangular fuzzy numbers [in general (a, b, c)] in accordance with the condition shown in (7) in the universe of discourse [0, 1]. It is natural to consider that the elements of LT₁ are defined by the following fuzzy numbers: AF = (0, 0, 0.05, 0.1), MF = (0.1, 0.2, 0.3), QT = (0.2, 0.3, 0.4), SF = (0.3, 0.4, 0.5), NS = (0.4, 0.5, 0.6), ST = (0.5, 0.6, 0.7), PT = (0.6, 0.7, 0.8), QF = (0.7, 0.8, 0.9), MT = (0.8, 0.9, 1), AT = (0.9, 0.95, 1).² According to the definition in (7), the expected (average) values of these fuzzy numbers are: E(AF) = 0.039, E(MF) = 0.01, E(QT) = 0.2, E(PF) = 0.3, E(SF) = 0.4, E(NS) = 0.5, E(ST) = 0.6, E(PT) = 0.7, E(QF) = 0.8, E(MT) = 0.9, and E(AT) = 0.961.

If so some of the above linguistic truth-values are assigned to the propositions in P₁ (instead of assigning the numerical truth-values as it is shown in the above), the following formulation can be found reasonable, LinguisticTruthValue(p₁) = E(AF) = 0.039, LinguisticTruthValue(p₂) = E(AF) = 0.039, LinguisticTruthValue(p₃) = E(MF) = 0.1, LinguisticTruthValue(p₄) = E(QT) = 0.8, LinguisticTruthValue(p₅) = E(QF) = 0.2. Note that there is a slight difference between the numerical truth-values assigned before and the numerical truth-values calculated this time. Before it was TV₁ = (0, 0, 0.1, 0.8, 0.2) and now it is TV₁ = (0.039, 0.039, 0.1, 0.8, 0.2).³

3. FORMAL SETTINGS OF GD

The formal expression of GD is given by the following 4-tuple vector:

\[
\Omega = ((P \models TV), (P_d \models TV_d), G, D). \quad (8)
\]

²For the illustrations of these fuzzy numbers, see Figure 4.
³From decision-making viewpoint, this difference is insignificant. This issue will be cleared if the measures shown in Section 4 are used to quantify the information content of these two formulations.
In the expression of Ω, P denotes a vector of propositions defined by (1)–(2); TV denotes the vector of truth-values defined by (4) or (7), i.e., TV = (TV_1, ..., TV_i, ..., TV_n) ∈ {TruthValue(p), LinguisticTruthValue(p_i)}, i = 1, ..., n; Size(P) = n; P_d denotes a proposition defined by (5); TV_d denotes the truth-value defined by (6) or (7), i.e., TV_d ∈ {TruthValue(p), LinguisticTruthValue(p_d)}; G: {[0,1] ^ n} → [0,1]; and D: [0,1] ^ 3 → [0,1]; “P |= TV” is the general opinion part and “P_d |= TV_d” is the desire, or requirement part. The function denoted by G measures the certainty compliance of Alternative(P) for Criterion(P) based on Quantifier(P), i.e., how clearly Alternative(P) is (considered to be) known. The function D measures the desire compliance of Alternative(P) for Criterion(P) based on Quantifier(P_d), i.e., how desirable Alternative(P) is or how strongly Alternative(P) fulfills the requirement.

Before introducing the equations of G and D, it is necessary to consider some semantics issues of a fuzzy proposition [Ullah and Monnet, 2002; Ullah, 2002, 2004]. These issues are explained by the following axioms: Local, Global, Granule, and Desire Definiteness Axioms. The first three axioms deal with P and its truth-values TV (i.e., deal with the general opinion part of GD) and the other deals with P_d and its truth-value, TV_d (i.e., deals with the desire or requirement part of GD). The next four subsections describe these axioms. The equations of G and D come automatically in the last two subsections, respectively.

3.1. Local Definiteness Axiom

Local Definiteness Axiom refers to the definiteness of Alternative(p_i) in terms of its truth-value TV_i, ∃i ∈ [1, ..., Size(P) = n]. If TV_i = 0 or 1, that means that p_i is completely true or false; the knowledge is complete regarding Alternative(p_i) for Criterion(p_i) on the basis of Quantifier(p_i). This is referred to as “local definite” information. On the other hand, if TV_i = 0.5, that means that p_i is neither true nor false; the knowledge is not at all complete regarding Alternative(p_i) for Criterion(p_i) on the basis of Quantifier(p_i). This is referred to as “local indefinite” information. Moreover, if TV_i = (0.0,5) ∪ (0.5,1), that means that p_i is partially true or false; the knowledge is partially complete regarding Alternative(p_i) for Criterion(p_i) on the basis of Quantifier(p_i). This is referred to as “partial local definite” information.

The information related to Local Definiteness Axiom refers to the entropy of fuzzy propositions. Thus, the following function (a tent map) can be used to measure the local definite, local indefinite, and partial local definite information.

\[ \pi(p_i) : [0, 1] \rightarrow [0, 1] \]

\[ TV_i \rightarrow \max \left\{ \min \left( \frac{TV_i - 0}{0.5 - 0}, \frac{1 - TV_i}{1 - 0.5} \right) \right\} \]

According to (9), for local definite information \( \pi(p_i) = 0 \), for local indefinite information \( \pi(p_i) = 1 \), and for partial local definite information \( \pi(p_i) = (0, 1) \).

3.2. Global Definiteness Axiom

Global Definiteness Axiom refers to the definiteness of Alternative(P) in terms of all TV_i, ∀i ∈ [1, ..., Size(P)]. If one of the truth-values is unit and others are zero, i.e., ∃i \• TV_i = 1 ∧ ∀j \• TV_j = 0, j ∈ [1, ..., Size(P)] = \{i\}, that means that Alternative(P) is clearly known from the viewpoint of Criterion(P) with respect to Quantifier(P). This is referred to as “global definite” information. If all truth-values are equal to 0.5, i.e., TV_i = 0.5, for all i = 1, ..., n, that means that Alternative(P) is completely unknown for Criterion(P) on the basis of Quantifier(P). This is referred to as “global indefinite” information. If the information is neither global definite, nor global indefinite, then it is “partial global definite” information.

Consider a function Π as follows:

\[ \Pi : [0, 1]^n \rightarrow [0, 1] \]

\[ \left((\pi(p_1), \ldots, \pi(p_n))\right) \rightarrow \sum_{i=1}^{n} \pi(p_i) \]

Π is able to measure all aspects of Global Definiteness Axiom. Particularly, if the information is global definite, then Π = 0; if the information is global indefinite, then Π = n; and if the information is partial global definite, then Π = (0, n).

3.3. Granule Definiteness Axiom

The Granule Definiteness Axiom refers to such definiteness of Alternative(P) that is (or should be) affected by Size(P) = n, i.e., by the number of quantifiers in P. Generally speaking, Size(P) depends on the importance or significance of Criterion(P)—the more important or significant the Criterion(P) is, the larger the Size(P) is. For example, consider P_2, where Alternative(P_2) = Alternative(P_1), Criterion(P_2) = Criterion(P_1), Quantifier(P_2) ⊆ Quantifier(P_1) (Quantifier(P_2) = {Poor, Fair, Average, Good, Excellent, Outstanding}), Size(P_2) > Size(P_1). This formulation indicates that
Safety (Criterion(P1) = Criterion(P2)) is given more importance in P2 than it is given in P1. Otherwise, it is not natural to see the information in more details, i.e., it is not natural to use more quantifiers. Now, the granular definiteness should increase with an increase in \( \text{Size}(P) \), if a large \( \text{Size}(P) \) helps produce more local definite information. Otherwise, granule definiteness should decrease with an increase in \( \text{Size}(P) \) or should remain the same. For example, if \( TV_1 = (0, 0, 0.1, 0.8, 0.2) \), \( TV_2 = (0, 0, 0.1, 0.8, 0.2, 0) \), then \( TV_2 \) has one more local define information (3 local definite information in \( TV_2 \) and 2 local definite information in \( TV_1 \)). Therefore, \( P_2 \) carries a piece of information that is more granule definite than that of \( P_1 \).

To measure the aspects of Granule Definiteness Axiom, \( \Pi \) should be modified so that the number of quantifiers i.e., \( \text{Size}(P) = n \), can play its role. Particularly, if is divided by \( n \), then the modified function is able to measures the aspects of the Local, Global, and Granule Definiteness Axioms altogether. Therefore, such a modified function is the equation of \( G \). As such,

\[
G: [0, 1] \rightarrow [0, 1],
\]

\[
\left( \Pi, n \right) \rightarrow \frac{1}{n} \sum_{i=1}^{n} \pi(p_i).
\]

(11)

According to (11), if the information is global definite [all \( \pi(p_i) = 0 \)], then \( G = 0 \); if the information is global indefinite [all \( \pi(p_i) = 1 \)], then \( G = 1 \). Moreover, \( G \) decreases if an increase in \( \text{Size}(P) \) helps produce more local definite information [i.e., if number of \( \pi(p_i) \) = 0 increases with the increase in \( \text{Size}(P) \)] and \( G \) increases or remains the same if an increase in \( \text{Size}(P) \) helps produce partial local define or local indefinite information [i.e., if number of non-zero \( \pi(p_i) \) increases with the increase in \( \text{Size}(P) \)].

For example, recall the cases of \( P_1 \) and \( P_2 \). For \( P_1 \),

\[
G = \frac{0 + 0 + 0.2 + 0.4 + 0.4}{5} = 0.2
\]

and for \( P_2 \),

\[
G = \frac{0 + 0 + 0.2 + 0.4 + 0.4 + 0}{6} = 0.133.
\]

This means that the Safety of Design X is quite clearly (not completely) known in terms of Poor, Fair, Average, Good, and Excellent and it is (Design X) is more clearly known (not completely yet) in terms of Poor, Fair, Average, Good, Excellent, and Outstanding (\( G(P_2) < G(P_1) \)). Here, one more quantifier (Outstanding) helps know about Design X more clearly. Needless to mention that Safety is more importance in \( P_2 \) than it is in \( P_1 \) since \( \text{Size}(P_2) > \text{Size}(P_1) \).

3.4. Desire Definiteness Axiom

Desire Definiteness Axiom refers to the definiteness of \( P_d \) in terms of its truth-value \( TV_d \), whether or not \( \text{Alternative}(P) \) fulfills the requirement set by \( P_d \) and what degree. Desire definiteness can be measured by calculating the distance between \( TV_d \) and maximal or minimal truth-values in \( TV \). If \( TV_d \) is equal to or more than the maximum truth-values in \( TV \), then the desire or requirement is completely fulfilled. Alternatively, if \( TV_d \) is equal to or less than the minimal truth-values in \( TV \), then the desire or requirement is not at all fulfilled. Therefore, to derive the equation for \( D \), consider two extreme (maximal and minimal) truth-values, \( TV_s \) and \( TV_t \), respectively, where, \( TV_s = \max(TV, \forall i \in \{1, \ldots, n\}) \) and \( TV_t = \min(TV, \forall i \in \{1, \ldots, n\}) \). As such, Quantifier(\( p_s \)), where \( p_s \rightarrow TV_s, \exists p_t \in P \), is the serious point or the most appropriate characteristics of \( \text{Alternative}(P) \). Similarly, Quantifier(\( p_t \)), where \( p_t \rightarrow TV_t, \forall p_t \in P \), is the weak point or the most inappropriate characteristics of \( \text{Alternative}(P) \). For example, recall the case of \( P_1 \). In this case, \( TV_s = \max(0, 0, 0.1, 0.8, 0.2) = 0.8 \), and \( p_r = \text{“Safety of Design X is Good”} \rightarrow TV_s \). Therefore, Quanti(\( p_s \)) = “Good” is the serious point of Design X. In other words, if someone is looking for a design with good safety features, then Design X is an absolutely preferred alternative. Similarly, \( TV_t = \min(0, 0, 0.1, 0.8, 0.2) = 0 \), and \( p_r = \text{“Safety of Design X is Fair”} \rightarrow TV_t \). Therefore, Quanti(\( p_t \)) = “Fair” is a weak point of Design X. (Poor is also a weak point of Design X, however.) In other words, if someone is looking for a design with fair Safety features, Design X is not at all a preferred alternative because “Safety of Design X is fair” is its weak point (i.e., TruthValue(Safety of Design X is Fair) = 0).

Therefore, \( D \) can be calculated by using the following function:

\[
D: [0, 1]^3 \rightarrow [0, 1],
\]

\[
(TV_s, TV_t, TV_d) \rightarrow \max \left( \frac{TV_s - TV_d}{TV_s - TV_t}, 1 \right) \cdot 0.
\]

(12)

According to (12), if the information is desire definite, then \( D = 0 \). In addition, if the information is desire indefinite, then \( D = 1 \). Moreover, if the information partial desire indefinite, then \( D < 1 \). For example, recall the case of \( P_1 \). In this case, \( TV_s = \max(0, 0, 0.1, 0.8, 0.2) = 0.8, TV_t = \min(0, 0, 0.1, 0.8, 0.2) = 0 \). If \( P_d \)
= P_d = “Safety should be more or less Good”, then TV_d = 0.894. As a result,

$$D = \max \left( \min \left( \frac{0.8 - 0.894}{0.8 - 0}, 1 \right), 0 \right) = 0,$$

that means that Design X is an absolutely desirable alternative. If P_d is changed to “Safety should be somewhat Excellent”, then TV_d = 0.447. As a result,

$$D = \max \left( \min \left( \frac{0.8 - 0.447}{0.8 - 0}, 1 \right), 0 \right) = 0.441,$$

which means that Design X is less desirable.

### 4. DECISION PROCESS

The measures associated with Ω [i.e., the ordered-pair (G, D)] can help make decisions. Particularly, (G, D) quantifies the information content or entropy of the information carried by Ω. If (G, D) = (0, 0), that means that the associated alternative is both completely known and completely desirable—no entropy or average information content. Otherwise, there is an amount of entropy or average information content given by (G, D). This implies that (G, D) represents the definiteness position of an alternative. Seeing the definiteness positions of a given set of alternatives, one can select the preferred alternative. Compare the cases shown in Figure 2. As seen from Figure 2, there are two definiteness positions (0.2, 0) and (0.3, 0.33) for two different alternatives for the same criterion and for the same set of quantifiers. These two definiteness positions implies that the first alternative is more clearly known compared to the other (compare G = 0.2 and G = 0.3) and the first alternative is more desirable compared to the other (compare D = 0 and D = 0.33). Therefore, the alternative corresponding to the definiteness position (0.2, 0) is the preferred alternative.

The above-mentioned decision process is applicable if the decision is made based on a single criterion. If the decision is made based on multiple-criteria (a more realistic decision situation), then a cluster of definiteness positions is formed. See Figure 3 wherein two clusters of definiteness positions are shown. In this case, the balance or coherency among the definiteness positions should play the key role in selecting the preferred alternative [Ullah, 2004]. In other words, if the definiteness positions of a given alternative for a set of criteria are away from the origin (G = 0, D = 0) and widely scattered, then it is not a balanced or coherent alternative. The lack of coherency occurs if the alternative is not clearly known for all criteria or partially known for some criteria and partially unknown for some other criteria or does not fulfill the requirements for some criteria or partially fulfills the requirement for some other criteria. To ensure the selection of an alternative having the highest coherency, an aggregation function is needed, as follows:

$$\chi: [0, 1]^5 \rightarrow [0, 5],$$

$$(\min(G_i), \max(G_i), \min(D_i), \max(D_i), \alpha) \rightarrow \min(G_i) + \max(G_i) + \min(D_i) + \max(D_i) + \alpha,$$

where \(\alpha = [\max(G_i) - \min(G_i)] \times [\max(D_i) - \min(D_i)].$$

In (12), \((G_i, D_i) \in [0, 1] \times [0, 1], \forall i \in \{1, \ldots, v\}, \) “\(v\)” is the number of criteria. The value of \(\chi\) is large if \((G_i, D_i)\) are away from the origin and widely scattered (i.e., not coherent). On the other hand, the value of \(\chi\) is small if \((G_i, D_i)\) are not away from the origin and not widely scattered (i.e., coherent). For example, compare the values of \(\chi\) for two clusters shown in Figure 3. Cluster 1 corresponds to \(\chi = 1.362\), and cluster 2 corresponds to \(\chi = 1.848\), which means cluster 1 is more coherent than cluster 2. The visual inspection also sug-
gests the same. This exemplification implies that $\chi$ is able to measure the balance or coherency of the points in a cluster (in the interval $[0, 5]$) of definiteness positions and the value of $\chi$ decreases with the increase in the degree of coherency.

5. APPLICATION

A JAVA™ based computing tool is developed to put GD into practice. The tool has some interfaces to formulate decision problem in accordance with $\Omega$. Figure 4 shows some of the main interfaces. As seen from Figure 4, the interfaces help input and edit a list of criteria, alternatives (text or graphic), quantifiers, and fuzzy numbers to define linguistic truth-values. To use the computing tool the designers first set the preparatory information (a criterion, a set of alternatives, and the list of quantifiers, and the set of linguistic truth-values). The designers are then allowed to input appropriate linguistic truth-values to each proposition in general opinion part. At the same time, the designers can set the quantifier(s) and index for the desire or requirement part. Needless to say, once the truth-values of the propositions associated with the general opinion are assigned, the truth-value of the desire proposition is automatically assigned. Finally, the computing tool calculates the values of G and D for each alternative and plots the definiteness position, (G, D).

![Figure 4. Interfaces of GD-based computing tool.](image-url)
To demonstrate the performance of GD, recall the conceptual design problem shown in Figure 1. As seen from Figure 1, the design problem is to find out the preferred conceptual design to design a chair with movable back and leg supports for comfortable and efficient office-work. Figure 4 shows the proposed conceptual design alternatives: (1) sprocket-chain mechanism based conceptual design; (2) gear-rack mechanism based conceptual design; and (3) worm-gear mechanism based conceptual design. These alternatives are taken from the design scenario shown in Figure 1. Figure 5 shows the results: how clearly these three alternatives are known and how desirable the alternatives are in terms of G and D. The humanistic reasoning that underlies the results shown in Figure 5 is as follows.

The criteria “operational safety” and “simplicity” are considered to be the criteria for evaluation and “operational safety” is considered to be more important than “simplicity.” Based on this judgment, five quantifiers (very high, high, moderate, low, and very low) can be considered for the criterion “operational safety” whereas three quantifiers (simple, moderately simple, and complex) can be considered for the other criterion (simplicity). Note that the number of granules (five for operational safety and three for simplicity) refers to the aspect of Granule Definiteness Axiom as defined in Section 3.

For a preferred conceptual design “operational safety” should be as high as possible (logically which means “very high or high”). The sense of this preference is as follows. If the operational safety is not high enough then the users might not be interested to buy the product and thus, from the designers’ side, it is not an appropriate approach to put less attention to the operational safety. This preference yields $P_{d1} = P_{d2} = \text{“operational safety should be very high or high.”}$. This humanistic reasoning underlies the settings as shown in Figure 5 in the desire part of the criterion operational safety. Now, worm-gear mechanism is able to provide self-locking ability if the appropriate lead angle is chosen. Therefore, it will provide high operational safety by locking the mechanism in case there is an accident. For other
two alternatives, it is not clearly known whether or not they are able to provide self-locking. This humanistic reasoning underlies the truth-values assigned to the general opinion part as shown in Figure 5 for the criterion “operational safety.” As such, worm-gear mechanism has the definiteness position at (0.247, 0.175), gear-rack mechanism has the definiteness position at (0.416, 0.303), and sprocket-chain mechanism has the definiteness position at (0.311, 0.92). Therefore, from operational safety viewpoint worm-gear mechanism is the preferred conceptual design because its definiteness position is close to the origin compared to the definiteness positions of other two alternatives.

Let us focus the other criterion—“simplicity” of the design. A preferred design should be “moderately simple” in the sense that if the design is too simple, then one might question the novelty of the design. Alternatively, if the design is complex then it might cost more. This humanistic reasoning implies that $P_{d} = P_{d1} =$ “simplicity should be moderately simple”. This underlies the settings of the desire part shown in Figure 5 for the criterion “simplicity.” Now, simplicity of all alternatives is somewhat unknown to the designers because there is no straightforward evidence where either of the mechanisms is used for a chair. However, sprocket-chain mechanism seems simple more than it seems moderately simple or complex. Again, the gear-rack mechanism seems complex more than it seems moderately simple or simple. On the other hand, worm-gear mechanism seems neither simple nor complex. This kind of humanistic reasoning underlies the truth-values assigned to the general opinion part shown in Figure 5 for the criterion “simplicity.” As such, worm-gear mechanism has the definiteness position at (0.4, 0), gear-rack mechanism has the definiteness position at (0.385, 0.5), and sprocket-chain mechanism has the definiteness position at (0.426, 0.465). Therefore, from a simplicity viewpoint, the worm-gear mechanism is the preferred conceptual design because its definiteness position is close to the origin compared to those of other two alternatives.

Using the definiteness positions shown in Figure 5, the definiteness clusters for two alternatives are constructed, which are as shown in Figure 6. Accordingly, the coherency scores for three alternatives are determined. The results are as follows: $\chi = 2.174$ for the sprocket-chain mechanism, $\chi = 1.611$ for the rack-gear mechanism, and $\chi = 0.849$ for the worm-gear mechanism. As such, the conceptual design alternative with the worm-gear mechanism is the preferred alternative. The above result also implies that the worm-gear alternative is not the ideal conceptual design. There might be other alternatives whose definiteness positions might fall into the box of “ideal solutions” as shown in Figure 6. In other words, there is scope for searching for better conceptual design alternatives. It is worth mentioning that if an alternative remains preferred for each individual criterion, it has a high chance to be preferred as a whole (high possibility of getting low $\chi$), as it happens for the example of worm-gear mechanism. It is, however, a possibility, not necessarily be true for all cases.

6. DISCUSSIONS

The approach of “compliance analysis, then ranking analysis” is a desirable approach for both fuzzy [Smith and Verma, 2004] and nonfuzzy [Ullman, 2003] decision models when decision-relevant information is incomplete, imprecise, and evolving—a common scenario in conceptual design. The presented model does the same but from a different perspective, which is as follows. Even though an alternative is considered to be highly desirable (D is very low) but the alternative is not clearly known (G is very high), it is actually not a preferable alternative. In other words, the abundance or lack of knowledge and the desirability are collectively responsible for making the final decision.

The presented model is able to accept solely linguistic terms as input information. Thus, it would be easy to review the results at a later period of time. This, in turn, increases the possibility of effective communicates among the individuals directly or indirectly related to a design project. Such a transparency in organizing and communicating design information is one of the prerequisites for developing better-engineered tools for conceptual design [Hsu and Liu, 2000; Wang et al., 2002].
Determination of the importance of criteria is a cumbersome task for a user to perform in any multiple-criteria decision model [Shamsuzzaman, Ullah, and Bohez, 2003; Yager, 2004]. The presented model shows a relatively easy means to deal with this issue. Particularly, the concept of granule definiteness and the function $G$ take into account the importance of criteria without being given any extra work to the user. Moreover, the presence of fine classes (very high, high, moderate, low, very low) for some criteria and coarse classes (high, moderate, low) for some other criteria do not produce any computational complexity in the decision making process, as it does for other multi-granular decision models [Herrera, Herrera-Viedma, and Martinez, 2000].

There are many aggregation functions developed so far to aggregate the payoffs of multiple-criteria into a single decision score, such as logical aggregation functions ($t$-norm $s$-norm, parametric $t$- and $s$-norms), algebraic aggregation functions (arithmetic average, geometric average, weighted average), and hybrid aggregation function like Werner’s “fuzzy and” function [Scott and Antonsson, 1998; Moulianitis, Aspragathos, and Dentsoras, 2004]. In practice, the selection of the appropriate aggregation function remains a difficult task to perform [Scott and Antonsson, 1998; Moulianitis, Aspragathos, and Dentsoras, 2004; See, Gurnami, and Lewis, 2004]. For the presented decision model it is argued that a function that is able to capture the coherency of definiteness positions is the appropriate aggregation function. Therefore, the aggregation functions usually found in the literature [Scott and Antonsson, 1998; Moulianitis, Aspragathos, and Dentsoras, 2004; See, Gurnami, and Lewis, 2004] are somewhat inappropriate for the GD-based decision model because of their inabilities to capture the coherency of definiteness positions. For this reason a new aggregation function is required for the GD-based decision process, which is achieved by introducing the function $\chi$. However, this aggregation function is perhaps one of the possibilities. Further investigations can be carried out to develop other possible aggregation functions.

The computing tool shown in the last section can be improved further so that it can do both compute numerical truth-values from linguistic variables and from the voting of design team members. This will make the tool more useful for performing collaborative conceptual design.

7. SUMMARY AND CONCLUSIONS

A structured but linguistic form of information called “General-Opinion and Desire,” or GD, is presented to help make decisions while performing conceptual design. The structure of GD is defined by $\Omega = ((P = TV), (P_\ell = TV_\ell), G, D)$, where $P$ is a vector of propositions (containing a unique alternative and criterion and a set of quantifiers), $TV$ is the vector of truth-values $TV = (TV_i | i = 1, \ldots, \text{Size}(P) = n, \forall TV_i \in [0,1])$, $P_\ell$ is a proposition by which the desire or requirement is expressed, $TV_\ell$ is the truth-value of $P_\ell$, i.e., $TV_\ell \in [0,1]$; $G: [0,1]^n \rightarrow [0, 1]$, and $D: [0, 1]^3 \rightarrow [0, 1]$. $G$ measures the certainty compliance of an alternative for a criterion based on a set of quantifiers—how clearly the alternative is (considered to be) known. $D$ measures the desire compliance of an alternative based on the requirement or desire expressed by $P_f$—how strongly the alternative fulfills the requirement or desire. The ordered-pair $(G, D)$, referred to as definiteness position, is a significant piece of information for making decisions. $(G, D) = (0, 0)$ means that the associated alternative is completely known and it is completely desirable—no entropy or average information content. Otherwise, there is a certain amount of entropy quantified by $(G, D)$. Therefore, seeing the definiteness positions of a given set of alternatives, one can select the preferred alternative, i.e., the alternative closest to the origin $(G = 0, D = 0)$. If the definiteness positions of an alternative with respect to multiple criteria are considered, a cluster of definiteness positions is formed. In this case, the balance or coherency among the definiteness positions should play the key role in selecting the preferred alternative. To ensure the selection of an alternative having the highest coherency, an aggregation function is needed. The characteristic of such a function is that its value is large if the definiteness positions are away from the origin and widely scattered. A function $\chi: [0, 1]^5 \rightarrow [0, 5]$ is developed to quantify the coherency. A JAVA™-based computing tool is also developed to put GD into practice. Using the case of a real-life conceptual design problem (conceptual design of a chair having movable back and leg supports for efficient and comfortable office work), it is shown that the proposed decision model is useful in selecting the preferred conceptual design from a given set of alternatives. As the presented model is able to accept solely linguistic terms as input information, it would be easy to review the results and communicate the design results to other individuals directly or indirectly related to the design project. Such a transparency in organizing and communicating design information is one of the prerequisites for developing better-engineered tools for conceptual design. Nevertheless, there are some computing issues that need further investigations, such as the issue of automatic calculation of truth-values from the linguistic variables, from the votes of the members of a design team, and likewise.
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REFERENCES


Dr. A.M.M. Sharif Ullah is currently an Assistant Professor in the Department of Mechanical Engineering at UAE University. He was a faculty member in the Industrial Systems Engineering Program at Asian Institute of Technology, Thailand. He received his B.S. from BUET, Dhaka in 1992 and M.S. and Ph.D. from Kansai University, Osaka in 1996 and 1999, respectively. He is a senior member of SME and a member of AAAI and IPMM. His research interest is Intelligent Design and Manufacturing.